Portfolio Trading in Asynchronous Markets

Michael G Sotiropoulos

Global Equities Quantitative Research
Deutsche Bank

CFEM Financial Engineering Seminar
13 October 2016
Introduction

Single Market
  Optimal Liquidation
  Overnight Risk

Asynchronous Markets
  Definitions
  Assumptions and Method
  Lead-lag Effects
  Application to Two Overlapping Markets

Summary

References
Electronic trading algorithms are typically divided into two categories, based on the amount of discretion they can exercise.

1. Execution or liquidation
   - Asset, trade side, and quantity are predetermined by client/user
   - Speed of trading and venue allocation are the main algorithmic controls
   - Liquidity is consumed, trading is unidirectional (either buy or sell)
   - Done in either agency or principal capacity
   - Performance is measured against a conventional benchmark price

2. Systematic or market making
   - There is discretion on asset, trade side, quantity, and quote/limit price
   - Liquidity is provided, trading can be bidirectional
   - There is inventory risk besides execution risk
   - Done mostly in principal capacity
   - Performance is the sum of trading P&L and inventory MTM P&L

Here we focus on execution algorithms of large portfolios. The concepts are also applicable to systematic trading with large inventories.
Trading algorithms make decisions at two levels.

1. Strategic
   ▶ Manage the trade-off between price impact and risk
   ▶ Create an initial trading schedule based on urgency or risk aversion
   ▶ Revise the trading schedule based on market and execution conditions

2. Tactical
   ▶ Track and consume short term signals
   ▶ Create and manage child orders (quantities, limit prices)
   ▶ Allocate child orders across trading venues (order routing)

The risk-cost trade-off occurs at the strategic level and it is typically formulated as an optimization problem.

Our goal is to define the objective function for portfolio liquidation in the presence of markets with overnight breaks and asynchronous trading sessions.
The optimal portfolio liquidation problem is set up as follows:

- At time $t_0$ a portfolio has initial positions in $N$ assets, collected in a vector $x(t_0)$
- By time $T$ the portfolio is liquidated, therefore $x(T) = 0$
- The optimal holdings $x(t)$ lead to waves of trading orders of size $\Delta x(t) = x(t + \Delta t) - x(t)$.
- For one-sided liquidation, if $x_i(t_0) > (\leq) 0$ then $\Delta x_i(t) \leq (\geq) 0$

In practical applications, $\Delta t$ is usually between 1 and 30 minutes.

The typically non-linear, constrained optimization problem becomes:

$$\min_{x(t)} Z[x(t)]$$

subject to:

1. $x(T) = 0$ (2)
2. $g[x(t)] \leq 0$ (3)
3. $x_i(t_0) \cdot \Delta x_i(t) \leq 0$  $i = 1 \ldots N$ (4)

- $Z[x(t)]$ is the objective (utility) function, a functional of the trading trajectory
- $g[x(t)]$ are extra constraints (cash/beta/sector/country/style neutrality, etc.)
The objective function in the mean-variance framework is

$$Z [x (t)] = \frac{1}{2} \lambda R [x (t)] + C [x (t)] - \int_{t_0}^{T} x^T (t) \cdot \alpha (t) \Delta t. \quad (5)$$

- $\lambda$ coefficient of risk aversion
- $R$ aggregate P&L variance from holding positions $x (t)$ at each time $t$
- $C$ aggregate trading cost from trading orders $\Delta x (t)$ at each time $t$
- $\alpha$ expected exogenous price drift (alpha view) at time $t$

- Our focus here is the risk term $R [x(t)]$ under various market conditions
- We will use continuous time formalism for notational economy

Solutions to the above problem lead to portfolio implementation shortfall (PTIS) trading trajectories.
Nikkei 225 long-only portfolio; Notional = Net Value = 70.4 bln JPY.

Liquidation lasts for one full trading day.
Mean-variance optimization over ten equal trading periods.

**Figure:** PTIS liquidation schedule (left) and risk/cost trade-off (right).
Execution risk: the variance of the P&L of a strategy that holds $X$ shares at time $t_0 = 0$, and zero shares at time $T$.

- Arrival value: $\Pi(0) = Xp(t_0)$
- Terminal value $\Pi(T)$, all shares converted to cash
- $P&L = \Pi(T) - \Pi(0)$

Assuming arithmetic Brownian motion for the price $p(t)$ with constant volatility $\sigma$

$$R[x(t)] = \sigma^2 \int_{t_0}^{T} x^2(t) \, dt.$$  \hspace{1cm} (6)

Example: Single asset VWAP, continuous session

The trading trajectory is ($t_0 = 0$)

$$x(t) = X \left(1 - \frac{t}{T}\right).$$  \hspace{1cm} (7)

The risk term under constant price volatility $\sigma$ is

$$R_{VWAP} = \sigma^2 \int_{0}^{T} x^2(t) \, dt = \frac{1}{3} X^2 \sigma^2 T.$$  \hspace{1cm} (8)
**Overnight**: the strategy requires more than one trading day to liquidate $X$ shares.

- The market closes at time $T_c$ and reopens at time $T_o$.
- The execution risk for trading duration $T - t_0 > T_c - t_0$ becomes

$$R[x(t)] = \sigma^2 \int_{t_0}^{T_c} x^2(t) \, dt + \tilde{\sigma}^2 x^2(T_c) + \sigma^2 \int_{T_c}^{T} x^2(t) \, dt$$  \hspace{1cm} (9)

- $\tilde{\sigma}$ is the volatility of overnight (close-to-open) returns. Notice the units.

**Example: Single asset VWAP, one overnight halt**

The trading trajectory with one overnight break is ($t_0 = 0$)

$$x(t) = \begin{cases} 
X \left(1 - \frac{t}{T - T_o + T_c}\right), & 0 \leq t \leq T_c \\
X \frac{T - T_o}{T - T_o + T_c}, & T_c \leq t \leq T_o \\
X \frac{T - t}{T - T_o + T_c}, & T_o \leq t \leq T
\end{cases}$$  \hspace{1cm} (10)

The risk becomes

$$R_{VWAP} = \frac{1}{3} X^2 \sigma^2 (T - T_o + T_c) + X^2 \tilde{\sigma}^2 \left(\frac{T - T_o}{T - T_o + T_c}\right)^2$$  \hspace{1cm} (11)
The risk with one overnight break can be written as

\[ R_{VWAP} = \frac{1}{3} X^2 \sigma^2 (T - T_o + T_c) + X_c^2 \sigma^2 \]  

(12)
Easy to generalize over several overnight periods because overnight returns are assumed independent of intraday returns.

- Suppose that liquidation lasts for $M$ full trading sessions
- All trading sessions have equal duration, $\tau$
- Start at the beginning of the first trading session
- Finish at the end of the $M$-th trading session (go through $M - 1$ nights)
- Constant intraday volatility $\sigma$ and overnight volatility $\tilde{\sigma}$

At the beginning of each overnight halt $i$ we hold $X(i) = X \left(1 - i / M\right)$ shares

The overnight risk becomes

$$R_{O/N} = X^2 \sum_{i=1}^{M} \left(1 - \frac{i}{M}\right)^2 \tilde{\sigma}^2 = X^2 \left(\frac{(M + 1)(2M + 1)}{6M} - 1\right) \tilde{\sigma}^2$$

(13)

The total risk becomes

$$R_{VWAP} = R_{IDAY} + R_{O/N} = \frac{1}{3} X^2 \sigma^2 (M + 1) \tau + X^2 \left(\frac{(M + 1)(2M + 1)}{6M} - 1\right) \tilde{\sigma}^2$$

(14)

$$\lim_{M \to \infty} R_{VWAP} = X^2 M \left(\sigma^2 \tau + \tilde{\sigma}^2\right) / 3.$$
We need to estimate the overnight volatility \( \tilde{\sigma} \)

1. Brute force: from the variance of the close-to-open returns of each asset
2. Reduced form:
   - assume that the overnight variance is a fixed proportion of the daily (close-to-close) variance
     \[
     \tilde{\sigma}^2 = \alpha^2 \sigma_{CC}^2
     \]  
   - estimate a common \( \alpha^2 \) for groups of stocks, using overnight and close-to-close returns

\[
\begin{array}{|c|c|c|c|}
\hline
\text{Index} & \# \text{Stocks} & \text{Median } \alpha^2 & \text{Mean } \alpha^2 \\
\hline
\text{S&P 500} & 503 & 0.31 & 0.34 \\
\hline
\text{Russell 2000} & 1973 & 0.19 & 0.25 \\
\hline
\text{FTSE 100} & 100 & 0.25 & 0.26 \\
\hline
\text{EuroStoxx 600} & 600 & 0.26 & 0.28 \\
\hline
\text{Nikkei 225} & 225 & 0.40 & 0.42 \\
\hline
\text{Hang Seng} & 50 & 0.53 & 0.52 \\
\hline
\end{array}
\]

See also [1].
Two asset execution risk: Initial positions $X$ and $Y$ shares. The risk becomes:

$$R[x(t), y(t)] = \sigma_x^2 \int_{t_0}^{T_c} x^2(t) \, dt + \sigma_y^2 \int_{t_0}^{T_c} y^2(t) \, dt + 2\rho \sigma_x \sigma_y \int_{t_0}^{T_c} x(t) y(t) \, dt$$

$$+ \tilde{\sigma}_x^2 x^2(T_c) + \tilde{\sigma}_y^2 y^2(T_c) + 2\tilde{\rho} \tilde{\sigma}_x \tilde{\sigma}_y x(T_c) y(T_c)$$

overnight risk

$$+ \sigma_x^2 \int_{T_o}^{T} x^2(t) \, dt + \sigma_y^2 \int_{T_o}^{T} y^2(t) \, dt + 2\rho \sigma_x \sigma_y \int_{T_o}^{T} x(t) y(t) \, dt.$$

$\tilde{\rho}$ is the correlation of overnight returns.

Example: Two asset VWAP, one overnight halt

$$R_{VWAP} = \frac{1}{3} \left( X^2 \sigma_x^2 + Y^2 \sigma_y^2 + 2XY \rho \sigma_x \sigma_y \right) (T - T_o + T_c)$$

$$+ \left( X^2 \tilde{\sigma}_x^2 + Y^2 \tilde{\sigma}_y^2 + 2XY \tilde{\rho} \tilde{\sigma}_x \tilde{\sigma}_y \right) \left( \frac{T - T_o}{T - T_o + T_c} \right)^2.$$  \hspace{1cm} (17)

$N$-asset VWAP, one overnight halt

$$R_{VWAP} = \frac{1}{3} \mathbf{X}^T \mathbf{\Sigma} \mathbf{X} (T - T_o + T_c) + \mathbf{X}^T \tilde{\mathbf{\Sigma}} \mathbf{X} \left( \frac{T - T_o}{T - T_o + T_c} \right)^2.$$  \hspace{1cm} (18)
Typical configurations of asynchronous markets:

1. Overlapping, one common intraday interval (London/New York)
2. Non-overlapping, no common intraday intervals (Tokyo/New York)

Notation: given two markets $m$ and $n$, we label the intervals as

- $mn$ both markets are open
- $m\tilde{n}$ market $m$ is open, market $n$ is closed
- $\tilde{m}\tilde{n}$ both markets are closed, market $m$ closed first
Asynchronous Markets: Assumptions and Method (I)

Covariance of daily returns in asynchronous markets has been studied in the literature, see for example [2, 3, 4].

We make the following two assumptions:

1. When a market is closed, there exists a hidden stochastic process for the price that is observable only at the closing and opening times.

2. Returns are serially independent both during the trading session and when a market is closed. Serial independence also holds for unobservable returns.

Consequences of serial independence:

1. Portfolio variance within two time points is additive
   - The time line is broken up into subintervals as in the earlier figure
   - Correlations need to be estimated within each subinterval, i.e. $\rho \left( \tilde{t}_1 \tilde{t}_2 \right)$, $\rho \left( \tilde{t}_1 \right)$, $\rho \left( \tilde{t}_2 \right)$ and $\rho \left( 1 \tilde{t}_2 \right)$
   - Correlation risk is aggregated by summing subinterval covariances

2. We estimate subinterval correlation by using pairs of observable returns that overlap only within the subinterval
   - The fact that returns may have portions that go beyond the subinterval, does not bias the subinterval correlation, although it may increase its confidence interval (may reduce efficiency of estimation)
\( \rho(12) \) is estimated the same way as for synchronous markets. We correlate intraday returns over the common trading interval.

\( \rho(\tilde{1}\tilde{2}) \) is estimated from intraday returns in mkt 1 and overnight in mkt 2 (orange lines in fig.)

\( \rho(\tilde{1}2) \) is estimated from overnight returns in mkt 1 and intraday in mkt 2

\( \rho(\tilde{1}\tilde{2}) \) is estimated from overnight returns in mkt 1 and following overnight returns in mkt 2 (green lines in fig.)

\( \rho(\tilde{2}\tilde{1}) \) is estimated from overnight returns in mkt 2 and following overnight returns in mkt 1 (blue lines in fig.)
Correlations $\rho (\tilde{1}\tilde{2})$ and $\rho (\tilde{2}\tilde{1})$ capture lead-lag effects of one market (the one that closes earlier) on the other market (the one that closes later).

The overnight return of security $i$ trading in mkt 1 as of day $t$ is $\tilde{r}_{i,t}$. We can define two covariances of overnight returns:

$$\text{cov}_{ij}(\tilde{1}\tilde{2}) = \text{cov}(\tilde{r}_{i,t}, \tilde{r}_{j,t}) = \frac{1}{T} \sum_{t=1}^{T} \tilde{r}_{i,t} \tilde{r}_{j,t}$$ (19)

$$\text{cov}_{ij}(\tilde{2}\tilde{1}) = \text{cov}(\tilde{r}_{i,t}, \tilde{r}_{j,t-1}) = \frac{1}{T-1} \sum_{t=1}^{T-1} \tilde{r}_{i,t} \tilde{r}_{j,t-1}$$ (20)

and the corresponding correlations

$$\rho_{ij}(\tilde{1}\tilde{2}) = \frac{\text{cov}_{ij}(\tilde{1}\tilde{2})}{\tilde{\sigma}_i \tilde{\sigma}_j}, \quad \rho_{ij}(\tilde{2}\tilde{1}) = \frac{\text{cov}_{ij}(\tilde{2}\tilde{1})}{\tilde{\sigma}_i \tilde{\sigma}_j}$$ (21)
Case study: Create a portfolio that is long all the securities in the SPX and NKY indices. Measure the pairwise correlations in two ways

1. lag = 0; Japan leads US
2. lag = 1; US leads Japan

We report the average correlation for the following sets of returns

1. daily: close-to-close in both markets
2. overnight: close-to-open in both markets
   - at lag = 0 these are the green lines, and at lag = 1 the blue lines in the earlier figure.
3. occo: open-to-close in mkt 1 (NKY) and close-to-open in mkt 2 (SPX)
   - at lag = 1 these are the orange lines in the earlier figure

Results:

<table>
<thead>
<tr>
<th>lag</th>
<th>daily</th>
<th>overnight</th>
<th>occo</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>28%</td>
<td>32%</td>
<td>20%</td>
</tr>
<tr>
<td>1</td>
<td>38%</td>
<td>47%</td>
<td>34%</td>
</tr>
</tbody>
</table>

Evidence that the US market leads Japan.
Liquidation of $X$ shares of a security in mkt 1 (London) and $Y$ shares of security in mkt 2 (New York).

- Start time $t_0$, when both markets are trading
- End time $T$, with $T > T_{c,1}$ and $T > T_{c,2}$
- Arrival value in ccy 1 (GBP): $\Pi_0 = Xp_X(t_0) + QYp_Y(t_0)$
- FX rate $Q$ considered fixed (no EQ-FX risk assessed). Set $Q = 1$

There will be five contributions to the risk term, from the five distinct time intervals.
The variance of the P&L as of time $t_0$ becomes:

$$R [x (t) , y (t)] =$$

$$\sigma_x^2 \int_{t_0}^{T_{c,1}} x^2 (t) \, dt + \sigma_y^2 \int_{t_0}^{T_{c,1}} y^2 (t) \, dt + 2 \rho (12) \sigma_x \sigma_y \int_{t_0}^{T_{c,1}} x (t) y (t) \, dt$$

both mkts open

$$+ \sigma_y^2 \int_{T_{c,1}}^{T_{c,2}} y^2 (t) \, dt + 2 \rho (\tilde{1} \tilde{2}) \tilde{\sigma}_x \sigma_y x (T_{c,1}) \int_{T_{c,1}}^{T_{c,2}} y (t) \, dt$$

mkt 1 closed, mkt 2 open

$$+ \tilde{\sigma}_x^2 x^2 (T_{c,1}) + 2 \rho (\tilde{1} \tilde{2}) \tilde{\sigma}_x \sigma_y x (T_{c,1}) y (T_{c,2}) + \tilde{\sigma}_y^2 y^2 (T_{c,2})$$

both mkts closed

$$+ \sigma_x^2 \int_{T_{o,1}}^{T_{o,2}} x^2 (t) \, dt + 2 \rho (\tilde{1} \tilde{2}) \tilde{\sigma}_x \sigma_y y (T_{c,2}) \int_{T_{o,1}}^{T_{o,2}} x (t) \, dt$$

mkt 1 open, mkt 2 closed

$$+ \sigma_x^2 \int_{T_{o,2}}^{T} x^2 (t) \, dt + \sigma_y^2 \int_{T_{o,2}}^{T} y^2 (t) \, dt + 2 \rho (12) \sigma_x \sigma_y \int_{T_{o,2}}^{T} x (t) y (t) \, dt.$$
We proposed a simple extension of the standard covariance calculation for asynchronous markets.

- The main assumption is serial independence of returns even when markets have stopped trading.
- Variance aggregation is additive, but different correlations need to be estimated.
- No need to specify the time scaling of overnight volatilities (no $\sqrt{t}$ rule needed).

Finally, in the context of portfolio optimization, different risk aversion coefficients can be applied to different time intervals.


All opinions are the author’s personal views and not necessarily of Deutsche Bank AG, its affiliates, or subsidiaries.